Math 404 Term Paper

The following is a description of an *research/expository* paper that will be worth 15% of your final grade. You will be scored out of 100, depending on the quality of what you turn in. **The paper will be strictly graded** on the following criteria:

- Quality of presentation and writing: clarity, use of appropriate English syntax and grammar;
- Correctness: are the mathematics correct? Are they symbols and notation consistent and appropriate?
- Content and flow: does the paper actually make sense and does it read well? Is it compelling?
- Depth of research: how much work went into investigating the topic and formulating the paper? Does it appear that the author understands the mathematical concepts?

Note: this is a **term paper**, as such, you should be working on it throughout the semester; it will actually be graded strictly. If it is clear that little time was spent, or in-depth thought was not given to the research and exposition, then a zero is possibile. Do not expect to score highly if you don't master the major mathematical terms at play—and many of them will be intellectually challenging. (You are not expected to become experts in cutting edge mathematical research to write this paper; however, part of the challenge in selecting a topic is to find research references at the appropriate level, and then find a corresponding voice in which to write. If you struggle with this, see me; in fact, see me anyways.)

This paper will be due on Monday, December 1st, in class. NO EXTENSIONS.

Prompt: Write a (roughly) 12 page (double-spaced, including math content, not including references) paper that fleshes out any of the topics described below. In particular, I want you to read about the topic (from various legitimate sources), develop a baseline understanding, and then pursue further references to continue refining your understanding of the notation, theorems, techniques, and computations involved. I want you to demonstrate that you understand the Math 404 concepts in the topic of your choosing. Beyond this, you may be creative with your exposition. This paper must find a balance between exposition and mathematics, i.e., it cannot *just* be mathematics, nor can it *just* be explanatory/historical.

The following is just a rough suggestion of what you be covered in a paper, inside a given topic (some description, some math, and some qualitative analysis): Where does the PDE come from (physical modeling, or other branches of mathematics)? Why do we care about it (application/s)? How can it be solved, historically and in modernity? What techniques are typically associated with this type of problem? Do people simulate solutions? How?

Obviously, some prompts will be be directly aligned with all of the above questions (e.g., a prompt concerning a theory or technique won't necessarily be tied to one PDE; if you are studying one famous PDE, it may have many applications and many analytical approaches). You will have

to use some discernment in selecting your topic, and after preliminary research, deciding what will make a compelling paper. I recommend that each of you discuss (early on, before mid-semester) your prompt and an outline with me.

Topics: Below I have provided a list of topics that could be used for the prompt. These are meant to be general "springboards", and I have simply provided a few key words and phrases, as well as some links and references as starting points. Once you've chosen a topic, there is some flexibility with the prompt. And you can design your own prompt/topic.

- *General First Order Equations*—General theory of quasilinear existence and uniqueness; shocks, fans, the general method of characteristics; shock solutions; higher dimensions.

  Strauss Chapter 14, DuZ Chapter 7, Bell Chapter 4

- *Wave Equation in Higher Dimensions*—Solutions in 2-D and 3-D and their relationships; Huygen's principle; domain of dependence/influence; general formulae for solutions on nice domains (Kirchhoff); effects of nonlinearities.

  Bell Section 7.3, Strauss Chapter 9, Active Link 1, Active Link 2

- *Electricity and Magnetism/Maxwell's Equations*—Equations of electromagnetic phenomenon in 3-D; derivation; nonlinear variants.

  Strauss Chapter 13, Active Link 1, Active Link 2

- *The Drumhead*—Modeling and equations of displacements (linear) for general case; nice geometries and general geometries; eigenvalues and eigenfunctions.

  Strauss Chapters 10 and 13, Active Link 1, Active Link 2, Active Link 3

- *Schrodinger Equation*—Derivation (inspiration); applications and examples in 1, 2, and 3-D; time-independent and time-dependent.

  Strauss Chapters 9 and 13, Active Link 1, Active Link 2

- *Solve the Hydrogen Atom*—Derive solutions to the hydrogen atom using Schrodinger in 3-D; spherical harmonics; electron shells.

  Strauss Chapter 9, Active Link 1, Active Link 2

- *Beams and/or Elasticity*—Beam equation (possibilities include one or all of: Euler-Bernoulli, Rayleigh, Timoshenko), derivation of beam equation(s); nonlinear equations (large displacements); various boundary conditions; general solution. One could also consider 2-D (plates or shells) or 3-D elasticity (system of elasticity); tensor notation; general solutions; effects of nonlinearity.

  Bell Chapter 20, Active Link 1, Active Link 2, Active Link 3, Active Link 4

- *Fluid Dynamics*—Equations of fluid dynamics, Navier-Stokes Equations; compressibility; Millennium problem; notions of solution; well-posedness; Stokes flow; Euler's equations.

  Strauss Chapter 13, Active Link 1, Active Link 2, Active Link 3, Active Link 4

- *Diffusion and Randomness*—Connect the heat equation to random walks (in 1 or 2D) and/or Brownian motion; show connections between "random" phenomena and heat equation (and its solution, the fundamental solution).

  David Logan, Chapter 1.4, Active Link 1, Active Link 2, Active Link 3, Active Link 4

- *Black-Scholes Equation and Options Pricing*—The Black-Scholes equation and/or related variants as pertains to options pricing; derivation and explanation of terms and quantities, dynamics; actual implementation in modern finance (relation to the recession).

  Active Link 1, Active Link 2, Active Link 3,

- *Acoustics*—General principles of compressible gas dynamics in 3-D; sound propagation; effects of dissipation.

  Strauss Chapter 13, Active Link 1, Active Link 2

- *Gas Dynamics, Transonics, Shocks in 3-D*—Gas dynamics as pertains to flight; transonic flows; shock waves and shock solutions; nonlinear equations; higher dimensional considerations

  Strauss Chapter 14, Active Link 1, Active Link 2, Active Link 3, Active Link 4

- *Solitons/KdV/Water Waves*—Water wave equations; KdV equations; soliton solutions; general solution techniques; integrable systems.

  Strauss Chapter 14, Active Link 1, Active Link 2, Active Link 3

- *Flutter*—Equations of aeroelastic flutter; bifurcation; fluid-structure coupled system; applications in aerodynamics as well as human biology (snoring) and alternative energies (energy harvesting).

  Active Link 1, Active Link 2

- *Calculus of Variations*— Euler-Lagrange equations; infinite dimensional problems; function space; classical problems: isoperimetry, brachistochrone, tautochrone.

  Strauss Chapter 14, Active Link 1, Active Link 2, Active Link 3, Active Link 4

- *Finite Element Method*— Weak/variational form of a problem (e.g., elliptic); Sobolev spaces; meshes, basis, degrees of freedom; error estimates (Cea's Lemma); convergence; method of forcing.

  Active Link 1, Active Link 2

- *Machine Learning and Neural Networks in PDEs*—PDE solvers using NNs; related numerical methods for approximating functions

  Active Link 1, Active Link 2, Active Link 3,

- *Modeling and Analysis of PDEs in Sociological/Political/Epidemiological/etc. Applications*—PDE models and modeling for mult-agent interactive systems; mean-field game theory; spatially distributed group dynamics and evolutions

  Active Link 1, Active Link 2, Active Link 3,

- *Build your own prompt*— Discuss with me.

As with any research paper, in-text citations should be utilized, and a detailed bibliography present. (The style of annotation is not important to me, as long as it is consistent and one of the standard formats.) Forums and Wiki-based websites *do not constitute suitable sources for citation*, although you can certainly use them to get started. *Plagiarism of any sort—in part or whole—will not be tolerated and will result in a zero—this includes improper use of AI; I will check.*