Initial / Boundary Values

Let's solve on $\tau \in [t_0, t]$

$$\ddot{x} = -9.8$$

$$\int_{t_0}^{t} \frac{d}{d\tau}\left[\frac{d}{d\tau}x\right] d\tau = \int_{t_0}^{t} (-9.8)\, d\tau$$

FTC

$$\dot{x}(t) - \dot{x}(t_0) = -9.8\,[t - t_0]$$

Integrate again on $[t_0, t]$ + FTC

$$[x(t) - x(t_0)] - \dot{x}(t_0)[t - t_0] = -\frac{9.8}{2}[t - t_0]^2$$

Rewrite

$$x(t) = x(t_0) + \dot{x}(t_0)[t - t_0] - 4.9\,[t - t_0]^2$$

We would know the exact position if

we know: t_0, $x(t_0)$, $\dot{x}(t_0)$

When we solve a second order problem, we "expect" two "constants of integration"

If we provide two additional pieces of "data" – ICs or BVs, we can often uniquely determine the $sol^{\underline{n}}$.

Mutatis mutandis for n^{th} order problems.

An n^{th} order IVP or BVP is an n^{th} order ODE

$$F(x, y', y'', \dots, y^{(n)}) = 0 \quad x \in I$$

supplemented by n auxiliary conditions for y and its derivatives on I.

For instence: for $x_0 \in I$,

$$y(x_0) = y_0,\ y'(x_0) = y_1, \dots,\ y^{(n-1)}(x_0) = y_{n-1}$$

Let's consider the IVP

$$\begin{cases} y' = y^2 & x \in (0,\infty) \\ y(1) = -1 \end{cases}$$

Solution? $y(x) = -1/x$

We need to check TWO ITEMS:

$$\frac{dy}{dx} = [-1/x]' = -\left(-\frac{1}{x^2}\right) = \frac{1}{x^2} = \left(-\frac{1}{x}\right)^2$$
$$= y^2$$

So the ODE is satisfied on $(0,\infty)$.

$y(1) = -1/1 = -1$ ✓ So the IVP is satisfied. ← Compete sentence!

We know $x(t) = c_1 \cos(t) + c_2 \sin(t)$ solves $\ddot{x} + x = 0$. If we add two ICs:

$x(0) = 0$, $\dot{x}(0) = 1$, then

$0 \overset{!}{=} x(0) = c_1$ and $1 \overset{!}{=} -1 \cdot 0 + c_2 \cdot 1$

so $x(t) = \sin(t)$ solves the IVP.