

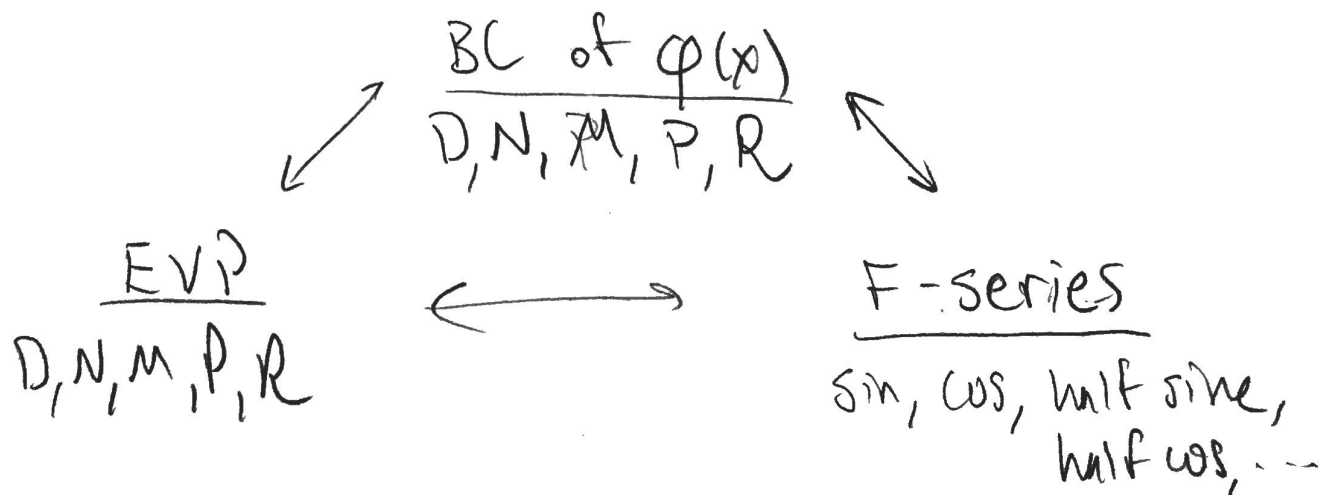
Fourier II

①

So the point is: given any $\varphi \in L^2(a,b)$
(like $\varphi(x) = x$ or $\varphi(x) = x^2$).

You can compute its Fourier series
in terms of any set of eigenfunctions
you like.

For compatibility, you want to make
sure the Fourier series you choose
corresponds to the BCs you have.



(2)

Two examples

$$\underline{\varphi(x) = 1}$$

F-cos series: $A_n = (\varphi, S_n)$ for

$$S_n = \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi}{L}x\right) \quad n=1, 2, 3, 4, \dots$$

$$A_n = (\varphi, S_n) = \begin{cases} \left(\frac{1}{\sqrt{L}}, 1\right) = \sqrt{L} & n=0 \\ \left(\sqrt{\frac{2}{L}} \cos\left(\frac{n\pi}{L}x\right), 1\right) = 0 & n=1, 2, 3 \end{cases}$$

$$\begin{aligned} \varphi(x) &= \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) = A_0 S_0(x) + 0 + 0 + \dots \\ &= \sqrt{L} \frac{1}{\sqrt{L}} = 1 \end{aligned}$$

F-sin series: $A_n = (\varphi, S_n)$ for

$$S_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) \quad n=1, 2, 3, \dots$$

$$A_n = \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right), 1\right)$$

$$= \begin{cases} -\frac{\sqrt{2L}}{\pi n} \\ 0 \end{cases}$$

n odd

n even

(3)

$$\phi(x) = 1 + \cos\left(\frac{7\pi}{L}x\right)$$

Which series is appropriate?

~~Sin~~ wrong BCs

cos ✓ what happens?

$$\phi(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right) \cdot \sqrt{\frac{2}{L}} + A_0 \frac{1}{\sqrt{L}}$$

where $A_n = \left(1 + \cos\left(\frac{7\pi}{L}x\right), S_n(x)\right)$

but orthonormality says:

$$A_n = 0 \quad \text{if } n \neq 1 \text{ or } 7!$$

so $\phi(x) = A_7 \sqrt{\frac{2}{L}} \cos\left(\frac{7\pi}{L}x\right) + A_0 \frac{1}{\sqrt{L}}$

$$A_7 = \left(\sqrt{\frac{2}{L}} \cos\left(\frac{7\pi}{L}x\right), \cos\left(\frac{7\pi}{L}x\right)\right)$$

$$\Rightarrow \phi(x) = 1 + \cos\left(\frac{7\pi}{L}x\right)$$

Amh! THIS IS IMPORTANT.