Basic Ideas + Classification

An ODE is an equation that involves expressions of a dependent variable (the solution), its derivatives, and one independent variable.

A PDE involves partial derivatives, since the dependent variable may change in various ways (more than one independent variable).

ODEs	PDEs
$\frac{dy}{dx} + y = y^2$	$u_{xy}^2 = u_{xx} + u_{yy}$
$\ddot{x} + \omega_0^2 x = F(t)$	$u_t = D \Delta u + F(x,t)$
$(y')^2 = (1+y)^{\sin(x)}$	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

The order of an ODE is simply the highest order derivative present. The highest derivative is dominant, and dictates the method and approach.

The general form of an n^{th} order ODE is

$$F\left(x, y, \frac{dy}{dx}, \ldots, \frac{d^n y}{dx^n}\right) = 0. \quad (y(x))$$

We can also write it in a suggestive way:

$$G(x, \dot{x}, \ldots, x^{(n)}) = f(t). \quad (x(t))$$

When (in this presentation) $f(t) \equiv 0$, the ODE is homogeneous.

(Different than what's in the book.)
(Not the best way to remember, either.)

Linearity / Nonlinearity

An ODE is linear if it is linear in terms involving the dependent variable. Nonlinear, otherwise.

An n^{th} order linear differential equation in $y(x)$ can be written as:

$$y^{(n)} + a_{n-1}(x)\, y^{(n-1)} + \ldots + a_2(x)\, y'' + a_1(x)\, y' + a_0 y = f(x)$$

or

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x) \frac{d^j y}{dx^j} = f(x)$$

(not the same as in the book)

In this presentation, the ODE is homogeneous if $f(x) \equiv 0$.